AN ENTIRE-DOMAIN BASIS GALERKIN'S METHOD FOR THE MODELING OF INTEGRATED MM-WAVE AND OPTICAL CIRCUITS

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ABSTRACT

The propagation characteristics of coupled rectangular dielectric waveguides are analyzed using the electric field integral equation. In contrast with the widely used subdomain basis Galerkin's method, in this work a novel set of entire-domain basis functions is utilized. This set consists of plane wave functions that satisfy Maxwell's equations in each guiding region. Computed dispersion curves are presented for a mm-wave transmission line and compare very closely to results of other techniques. The present implementation can also deal with integrated optical circuits and its main advantage is superior numerical efficiency.

1. INTRODUCTION

The rectangular dielectric waveguide (RDW) is the fundamental building block of integrated optical circuits [1]. It has also found many applications in mm and sub-mm wave integrated transmission lines [2]. The propagation modes in these structures are confined in the regions of the RDWs and can be investigated by means of methods such as mode matching techniques [3] and electric field integral equations (EFIE) [4,5]. The EFIE in its standard form considers the electric field in the RDWs domain as an equivalent polarization current. Boundary integral equation approaches have also been elaborated [6].

In this paper, the standard EFIE is employed and is subsequently solved using Galerkin's method. Most of the existing implementations of this method are based on the use of pulse subdomain basis functions [5], that require a fine segmentation of the guide cross-section. Consequently, a large number of unknowns is introduced, which renders the results either unstable or numerically costly [7]. Furthermore, from physical point of view, a subsectional expansion is not proper since the discontinuous variation of the electric field creates fictitious charges and currents on the boundaries of the subdomains.

In order to overcome the aforementioned drawbacks, a novel set of entire-domain basis functions appropriate for RDWs is developed. This set is constructed by discretizing an exact integral representation of the electric field inside each guiding region. The simple plane wave functions derived satisfy Helmholtz's equation, therefore representing a physical expansion mechanism, valid for both lossy and lossless materials. Even though the modeling of RDWs with entire-domain expansion terms has been suggested elsewhere [4,7,8], the systematic implementation of plane wave basis functions for the solution of the EFIE is believed to be new. Besides the accuracy and the simplicity of the proposed method, its major advantage lies in its numerical efficiency, since very satisfactory results can be obtained using only a few expansion terms. Waveguides of more complicated cross-section can also be analyzed, provided that they decompose into a number of RDWs.

As an application of the developed technique, numerical results in the form of propagation and attenuation constants are presented for the dominant and the first higher order mode of a mm-wave transmission line. The excellent agreement observed with previously published results [3] not only establishes the validity of the present work, but also reveals its computational efficiency.

2. THEORY

The waveguide structure considered in the analysis is depicted in Figure 1. It consists of L RDWs embedded in an infinite dielectric layer of permittivity $\varepsilon_{r0}\cdot\varepsilon_v$ (ε_v denotes vacuum's constant). This layer itself is part of a general planar stratified medium (not shown), with or without conducting ground planes. The whole structure is assumed to be uniform along the propagation axis y. The position of the i'th waveguide with width ℓ_x^i and height ℓ_z^i is defined by its cross-section centre coordinates (x_0^i, z_0^i), while its relative permittivity $\varepsilon_r^{(i)}$ is assumed constant. Losses are taken into account by allowing the dielectric permittivities to be complex in general. Finally, all the materials are characterized by the permeability μ_v of vacuum.

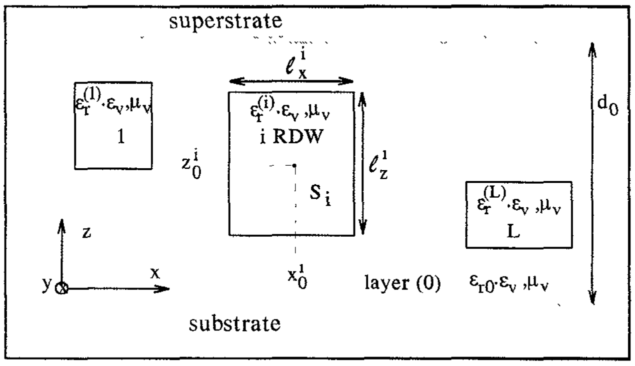

Figure 1: Cross-sectional view of L RDWs embedded in layer (0) of a planar stratified medium

The study of the propagation features of the structure of Figure 1 is based on the following rigorous integral representation [4] for the electric field

$$\overline{E}(\overline{r}) = k_v^2 \sum_{i=1}^{L} \delta\varepsilon_{ri} \iiint_{V_i} \overline{\overline{G}}(\overline{r},\overline{r}') \cdot \overline{E}(\overline{r}')\, dV' \qquad (1)$$

where $\delta\varepsilon_{ri} = \varepsilon_r^{(i)} - \varepsilon_{r0}$, $k_v = \omega\sqrt{\varepsilon_v \mu_v}$ and V_i is the volume occupied by the i'th RDW. The dyadic Green's function $\overline{\overline{G}}(\overline{r},\overline{r}')$ is represented in the Fourier-transform plane (k_x, k_y) and its spectral dyad $\overline{\overline{g}}(k_x, k_y; z, z')$ is computed using well known procedures [5]. The effects of the geometry of the layered dielectric surround are incorporated in a general manner into $\overline{\overline{g}}$.

We consider fields of the form $\overline{E}(\overline{r}) = \overline{e}(\overline{\rho})e^{-j\beta y}$ (for $e^{j\omega t}$ time dependence), where $\overline{\rho} = x\hat{x} + z\hat{z}$ and β is the propagation constant. A system of L coupled integral equations is then derived from (1), with unknown functions the field distributions $\overline{e}_i(\overline{\rho})$ (i=1,2,..,L) in the cross sections S_i of the RDWs. This system is numerically solved by employing Galerkin's method. To find an ideal mode expansion mechanism, we express $\overline{e}_i(\overline{\rho})$ in terms of its two dimensional Fourier transform [8] and impose on it the homogeneous Helmholtz equation for the i'th RDW:

$$\left(\nabla_{x,z}^2 + (k^{(i)})^2 - \beta^2\right)\overline{e}_i(x,z) = 0\,, \quad k^{(i)} = \omega\sqrt{\varepsilon_v \varepsilon_r^{(i)} \mu_v} \qquad (2)$$

This leads to the following condition between the spectral variables k_x and k_z

$$k_x^2 + k_z^2 = u_i^2(\beta), \qquad u_i(\beta) = \sqrt{(k^{(i)})^2 - \beta^2} \qquad (3)$$

Since k_x and k_z are not independent, the field $\overline{e}_i(\overline{\rho})$ is given by an inverse Fourier transformation of a single spectral variable, which is chosen to be the spectral angle φ_k, as

$$\overline{e}_i(x,z) = \int_0^{2\pi} \overline{C}_i(\varphi_k)\, e^{ju_i(\beta)\left[(x-x_0^i)\sin\varphi_k + (z-z_0^i)\cos\varphi_k\right]} d\varphi_k \qquad (4)$$

The above representation is rigorous and expresses the field in a RDW as a superposition of plane waves. Equation (4) can be utilized to construct entire domain basis functions for the unknown field $\overline{e}_i(x,z)$. As a first step, the integral in (4) is approximated by a finite summation over N_i angles φ_{kn} in the range $[0,2\pi]$, where $\varphi_{kn}=(n-1)\cdot 2\pi/N_i$, $n=1,2,\ldots,N_i$. Next we set

$$k_{xn}^i = u_i(\beta)\sin\varphi_{kn}\,, \quad k_{zn}^i = u_i(\beta)\cos\varphi_{kn} \qquad (5)$$

$$f_{in}(x,z) = e^{jk_{xn}^i(x-x_0^i)}\, e^{jk_{zn}^i(z-z_0^i)}\,, \quad \overline{C}_{in} = \overline{C}_i(\varphi_{kn}) \qquad (6)$$

Finally, the electric field in the i'th RDW is expanded in terms of N_i basis functions $f_{in}(x,z)$, as

$$\overline{e}_i(x,z) = \sum_{a=x,y,z} \sum_{n=1}^{N_i} \hat{a} C_{in}^a f_{in}(x,z)\,, \quad (x,z) \in S_i \qquad (7)$$

Equations (5-6) define a set of entire-domain basis functions, appropriate for RDWs. A remarkable feature of these functions is their explicit dependence on the wavenumber β, through the term $u_i(\beta)$ of equation (3).

Employing the above expansion terms to the Galerkin's solution of the EFIE, leads to an homogeneous linear system with unknown quantities the field coefficients C_{in}^{a} , i=1,2,...,L, n=1,2,..,N_i and a=x,y,z. The total number of unknowns is N_{tot}= $3(N_1+N_2+...+N_L)$ and the order of the matrix is $N_{tot} \times N_{tot}$. The propagation constant β is determined by requiring the vanishing of the system determinant. The elements of the system matrix are given by infinite spectral integrals of the variable k_x, which are computed numerically in conjunction with an efficient asymptotic extraction technique.

In constructing the set of basis functions (6), the Gauss law has not been used yet. If we impose this low on the field of equation (7), we obtain

$$\nabla \cdot \left(\bar{e}_i(x,z)e^{-j\beta y}\right) = 0 \Rightarrow \beta C_{in}^{y} = k_{xn}^{i} C_{in}^{x} + k_{zn}^{i} C_{in}^{z} \quad (8)$$

The above relation reduces the number of unknown field coefficients from $3N_i$ to $2N_i$ and the new system matrix is formed by applying the testing procedure with the two of the Cartesian components of the EFIE, as in [7].

3. RESULTS AND DISCUSSION

In order to validate the present technique and demonstrate its advantages, we analyze the structure of Figure 2. This shows a mm-wave monolithic transmission line [3] in which the guiding region consists of two (L=2) stacked RDWs. It should be noted that no EFIE solution of such a configuration has been reported in the literature. Some preliminary results of the present implementation are also given in [9].

Figure 3(a) depicts the dispersion diagram of this waveguide, for the dominant and the first higher order mode. Both modes are of E^z type, with dominant field components in the z direction. There are also included the results of the mode matching technique [3], with which excellent agreement is observed. The solid line corresponds to the dominant TM^z mode of the surrounding (referred to as the dominant substrate mode), which defines the lower limit for guided waves [5]. It is pointed out that the factor $u_1(\beta)$ in equations (2-5) may be real or imaginary depending on the relative values of $k^{(1)}$ and β. As a consequence, the expansion mechanism (6) for the first (i=1) waveguide consists of either plane waves or real exponential functions.

In Figure 3(b), the convergence of the propagation constant of the fundamental mode of Fig. 3(a) is plotted versus the number of basis functions N_1 and N_2 used in each waveguide, at the frequency of 125 GHz. The results obtained when $N_1 \geq 5$ and $N_2 \geq 10$ are practically indistinguishable. Convergence to within 0.01 percent is achieved using 5 basis functions for the first RDW (N_1=5) and 9 for the second one (N_2=9). In this case, the total number of unknowns is $3 \cdot (N_1+N_2)=42$ (28 if the Gauss law (8) is used) and the order of the system matrix is 42×42. These numbers are by far smaller than those required in an subsectional-basis Galerkin's solution of the EFIE. The efficiency of the present technique is laying on the fact that the expansion terms and the unknown electric field satisfy the same equations.

Finally, Figure 4 shows the frequency variation of the attenuation constant α (in dB per guided wavelength $\lambda g=2\pi/\beta$) of the dominant mode of Fig. 3(a), if losses in the dielectric materials are included. Again, very good agreement is observed between the present results and those of [3]. In this case, the factor $u_i(\beta)$ (i=1,2) is complex, resulting in complex basis function for both RDWs.

4. CONCLUSIONS

A new implementation of Galerkin's method has been developed for the integral equation analysis of rectangular dielectric waveguides. This is based on the use of entire-domain basis functions that satisfy Maxwell's equations inside each guiding region. Propagation characteristics are presented for a mm-wave transmission line and compare favourably to results of other well-established methods. The main advantages of this method are simplicity, accuracy and high numerical efficiency. The technique presented in this paper is now being employed for the study of field distributions, as well as propagation and leakage characteristics of integrated optical waveguides.

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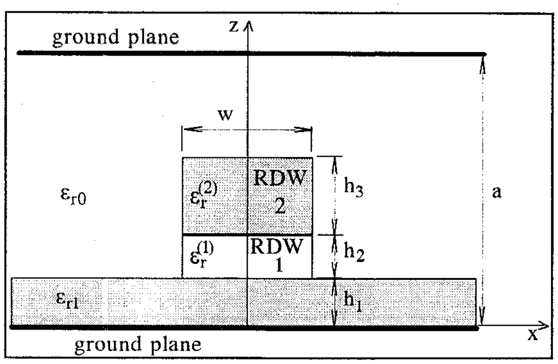
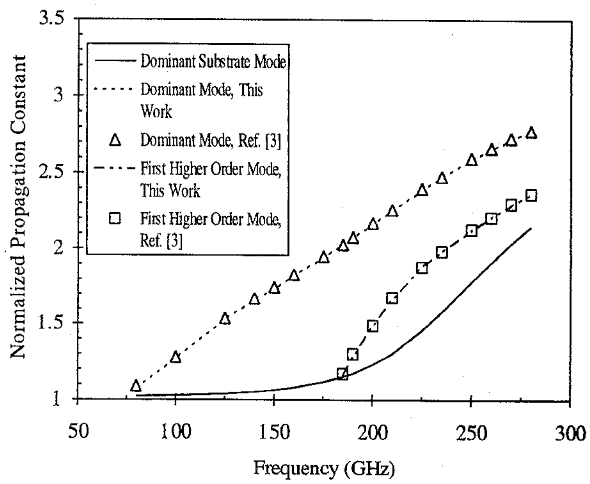
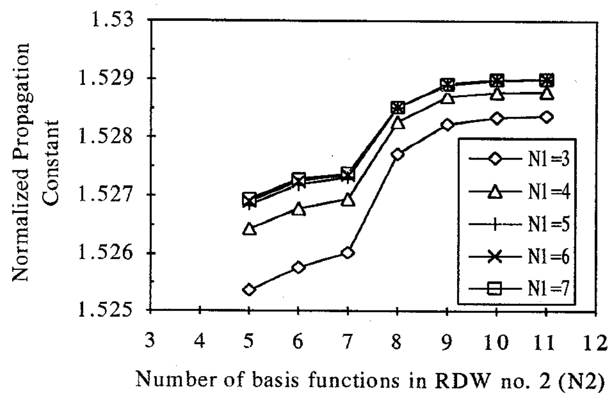


Figure 2: Geometrical configuration of a mm-wave transmission line [3] with parameters h_1=86.4 μm, h_2=130 μm, h_3=309 μm, a=0.231 cm, w=241 μm, $\varepsilon_{r1}=\varepsilon_r^{(2)}$=12.85 (GaAs) and $\varepsilon_r^{(1)}$=3. (polyamide).



(a)



(b)

Figure 3: Dispersion diagram of the structure of Fig. 2. (a) Comparison between the present method results and those of the mode matching technique [3]. (b) Convergence pattern of the fundamental mode of Fig. 3(a) at the frequency of 125 GHz.

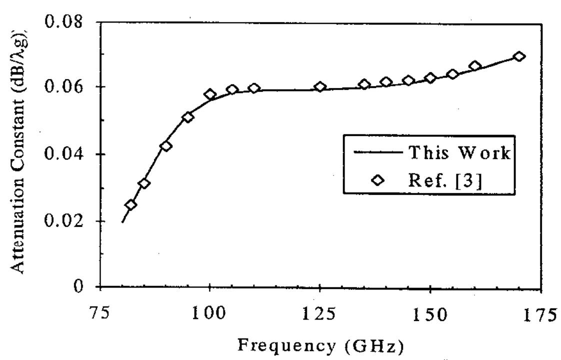


Figure 4: Attenuation constant as a function of frequency of the dominant mode of the waveguide shown in Fig. 2 with $\varepsilon_{r1}=\varepsilon_r^{(2)}$=12.85(1.0-j0.002) and $\varepsilon_r^{(1)}$=3.0(1.0-j0.001). Comparison with the results of [3].